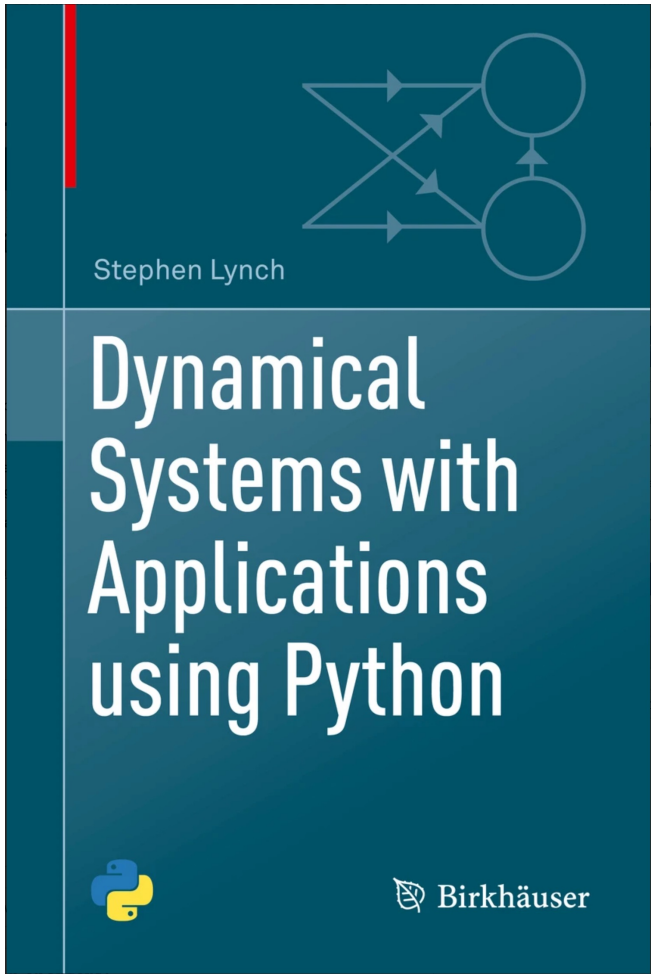
Dynamical Systems with Applications using Python

Dr Stephen Lynch NTF FIMA SFHEA





Stephen Lynch
ScholarGPS™ ID: 53907764717005

Highly Ranked Scholar - Lifetime:
#23 Dynamical system

AIMS

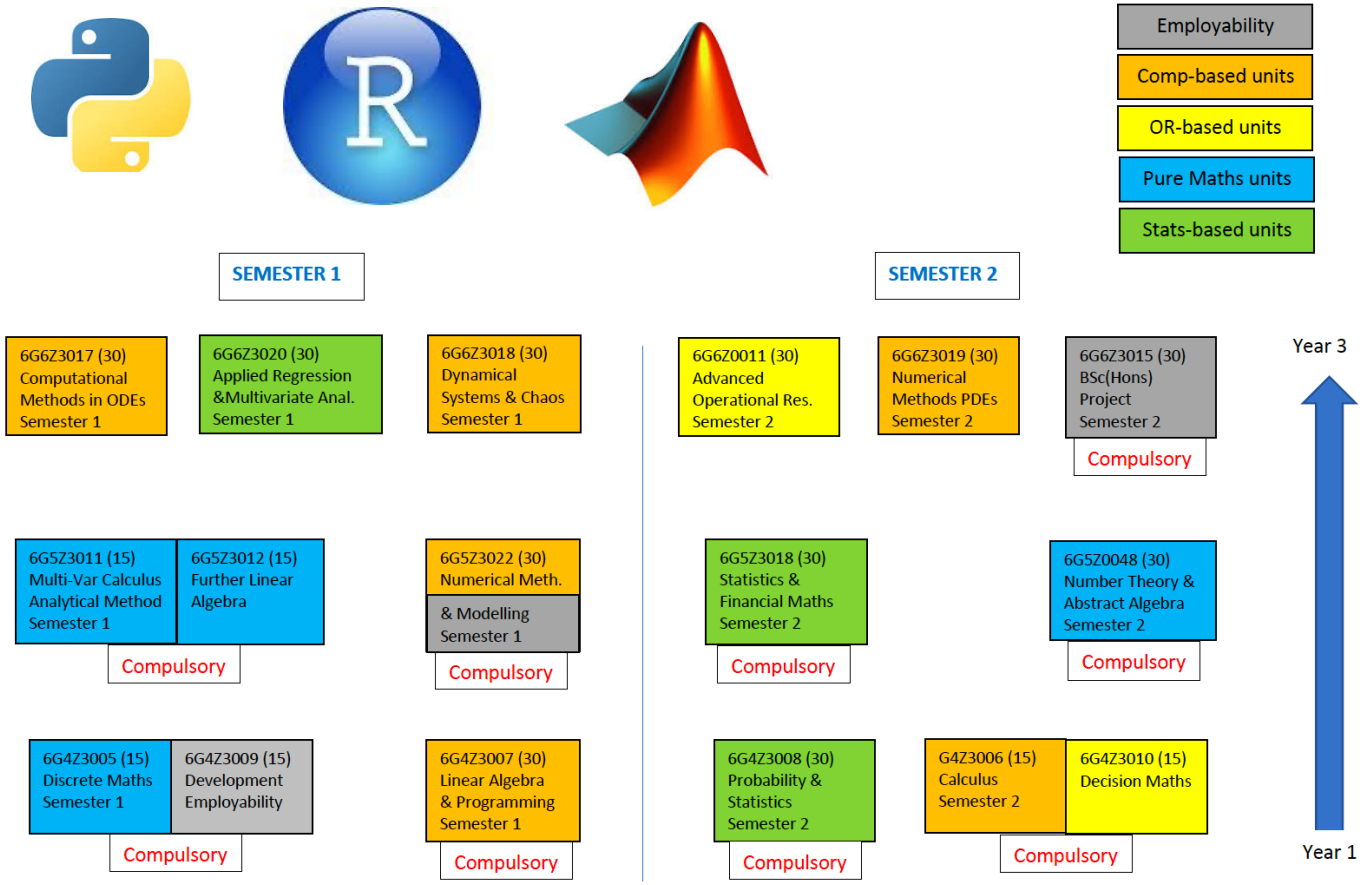
To explain how programming, computational modelling and simulation can improve the student experience.

OBJECTIVES

- To provide the tools for our students to become highly employable.
- To provide resources for students to use those tools.
- To provide extra- and co-curricula programming workshops for all staff and students across the university.



BSc(Hons) Mathematics Options Map 2023/2024





Student Union Teaching Awards
BSc(Hons) Mathematics shortlisted 8 times and won in 2012, 2018 & 2020!

Accessing Python (Open-Source Software)

There are many platforms, at MMU we use IDLE, ANACONDA and GOOGLE COLAB

1. https://www.python.org/downloads/



2. https://www.anaconda.com



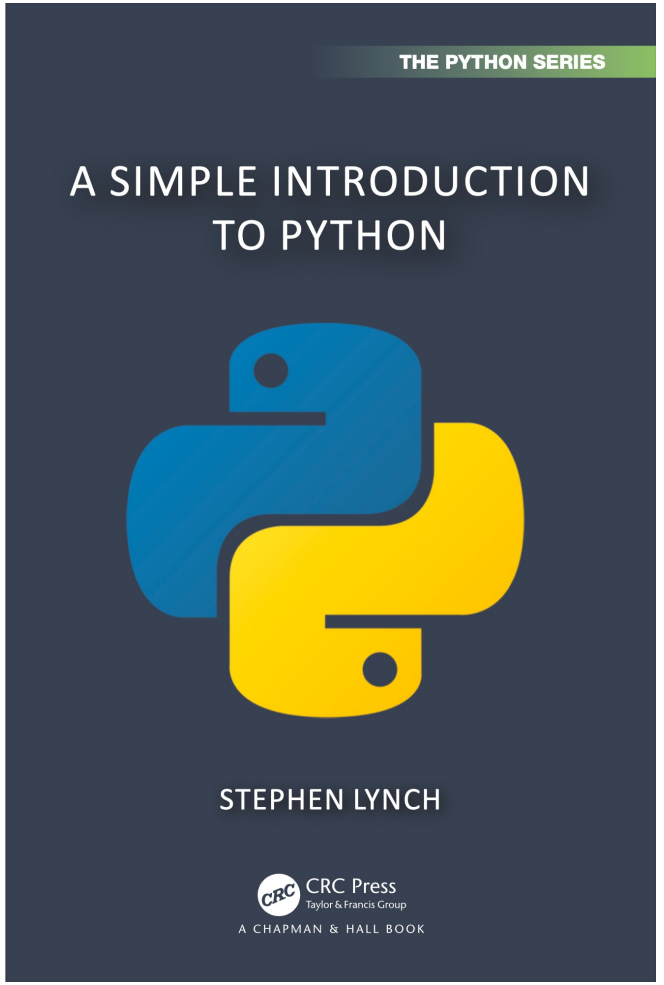
3. https://colab.research.google.com



FOUNDATION COMPUTING 2: Book Published June 2024, CRC Press (15 credit module – 1 hour lecture, 2 hours in the lab)

Week 1: Python as a Powerful Calculator
Week 2: Simple Programming with Python
Week 3: The Turtle Module and Graphics (Coursework Handed Out)
Week 4: NumPy and MatPLotLib
Week 5: Google Colab, SymPy and GitHub
Week 6: Python for Mathematics
Week 7: An Introduction to Cryptography
Week 8: An Introduction to Artificial Intelligence
Week 9: An Introduction to Data Science
Week 10: Object Oriented Programming

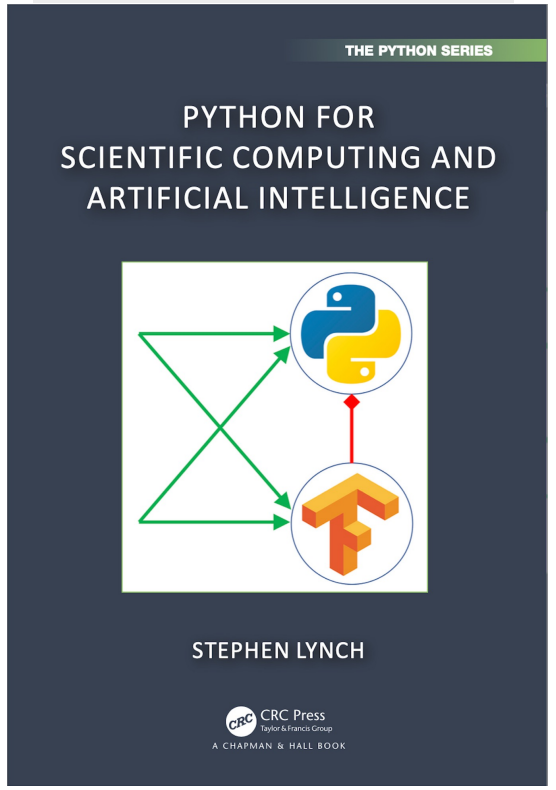
Week 11: Python in Computer Science and Mathematics at MMU



Python for Scientific Computing and TensorFlow for Artificial Intelligence

RISE
RISE.MMU.AC.UK

This 4-day interactive virtual workshop is designed for anyone in the Manchester Met community who wants to learn how to use Python for Scientific Computing and TensorFlow for Artificial Intelligence (AI) programming. No prior knowledge of Python or programming is required to benefit from this webinar series.



In A Nutshell...

- **Learn** ... to program in Python.
- **Apply** ... Python to Scientific Computing.
- **Solve** ... real-world problems.
- **Understand** ... how the brain works.
- **Upskill** ... to Artificial Intelligence (AI).
- **Earn up to 300 Rise points** ... which can be recognised within your degree.

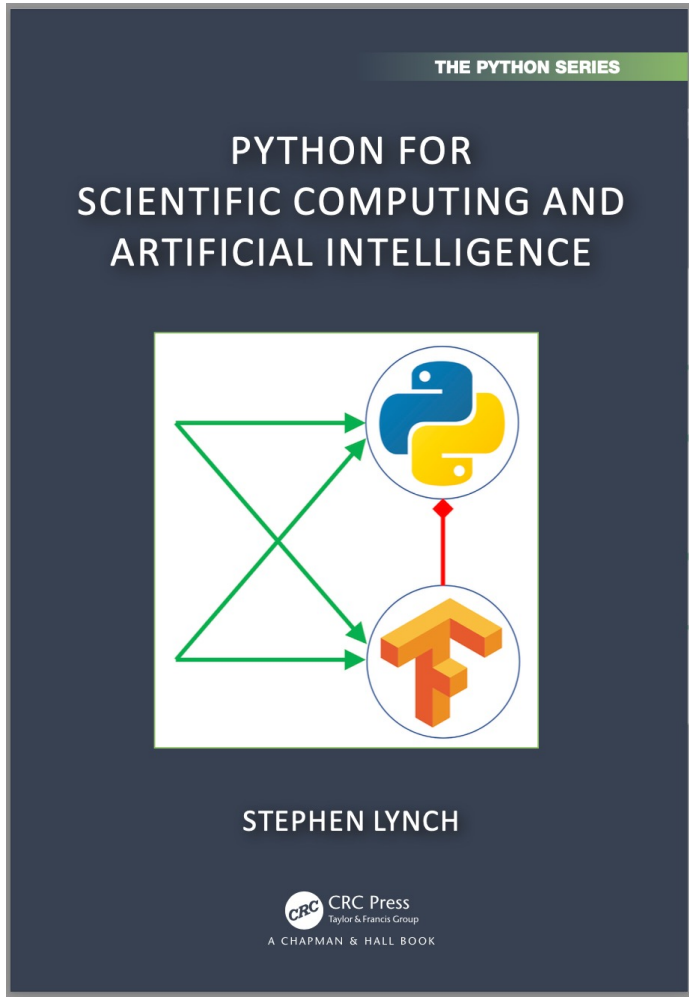
PYTHON for SCIENTIFIC COMPUTING and ARTIFICIAL INTELLIGENCE

SECTION 1: An Introduction to Python and Python for A-Level Maths

SECTION 2: Scientific Computing: Biology, Chemistry, Data Science, Economics, Engineering, Fractals and Multifractals, Image Processing, Numerical Methods for ODEs and PDEs, Physics and Statistics

SECTION 3: Artificial Intelligence: Binary Oscillator Computing, Neural Networks, Neurodynamics, Recurrent and Convolutional Neural Networks

CRC Press June 2023:
https://www.routledge.com/Python-for-Scientific-Computing-and-Artificial-Intelligence/Lynch/p/book/9781032258713#

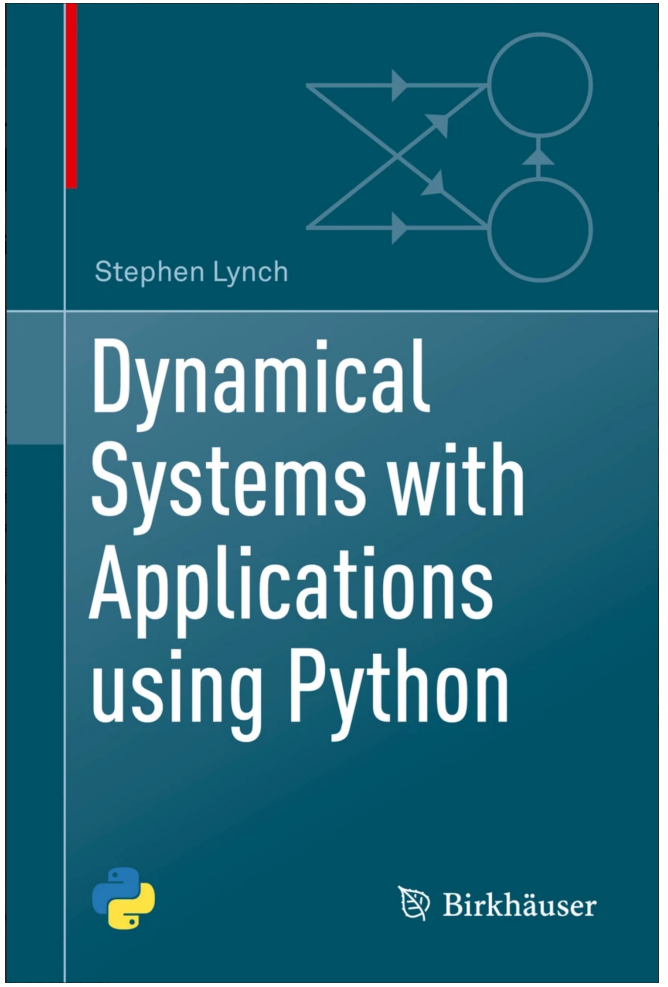


Year 3 MODULE: DYNAMICAL SYSTEMS AND CHAOS (30 Credits – 6 hours contact per week)

Students can use Python and/or MATLAB

Week 1: Separable & Linear ODEs and Phase Portraits
Week 2: Nonlinear Planar Systems
Week 3: Limit Cycles, Hamiltonian Systems and Poincaré Maps
Week 4: Lyapunov Functions and Stability
Week 5: Three-Dimensional Systems & Applications
Week 6: The Tent Map & Logistic Map and Population Dynamics
Week 7: Two-Dimensional Discrete Maps
Week 8: An Introduction to Artificial Intelligence
Week 9: The Ikeda Complex Iterative Map
Week 10: Fractals and Multifractals

Week 11: Revision



Year 3 MODULE: DYNAMICAL SYSTEMS AND CHAOS
(30 Credits – 6 hours contact per week)

Resources Available:

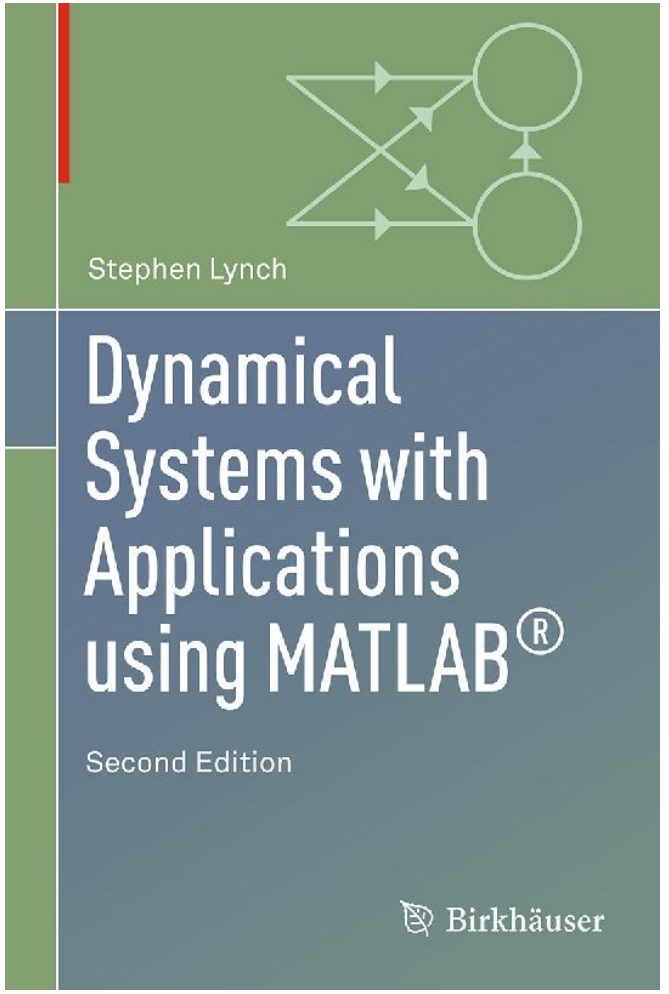
Python: Online Jupyter Notebook:

https://drstephenlynch.github.io/webpages/DSAP_Jupyter_Notebook.html

MATLAB Central File Exchange:

https://uk.mathworks.com/matlabcentral/profile/authors/63144

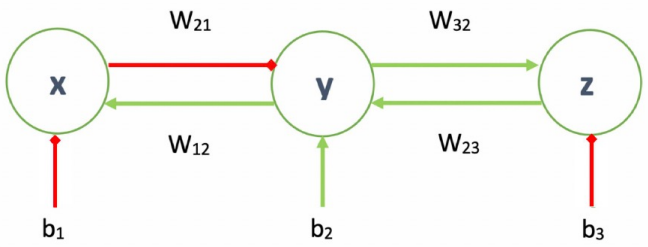
Third edition will be published in late 2024, Springer International Publishing.



Assessment

1CWK40: Four questions, two on discrete systems and two on continuous systems.

For example: Plot a bifurcation diagram for a 3-neuron module: (parameters and transfer functions given in the question).
[25 Marks]



1Exam60: Three-hour examination, answer 5 out of 8 questions. MATLAB is used as a powerful calculator and no programs are saved on the computer.

For example: Solve the IVP, $\frac{dr}{dt} = -\frac{r^3}{2}, \frac{d\theta}{dt} = 1, r(0) = 1, \theta(0) = 0$, and hence derive the Poincaré return map, $r_{n+1} = P(r_n)$, mapping points on the positive *x*-axis to itself. [10 Marks]

Week 1: Linear Planar Systems

```
# Program 03a: Phase portrait of a linear system.
# See Figure 3.8(a).

import matplotlib.pyplot as plt
import numpy as np
from scipy.integrate import odeint
import pylab as pl

# The 2-dimensional linear system.
a, b, c, d = 2, 1, 1, 2
def dx_dt(x, t):
    return [a*x[0] + b *x[1], c*x[0] + d*x[1]]

# Trajectories in forward time.
ts = np.linspace(0, 4, 100)
ic = np.linspace(-1, 1, 5)
for r in ic:
    for s in ic:
        x0 = [r, s]
        xs = odeint(dx_dt, x0, ts)
        plt.plot(xs[:,0], xs[:,1], "r-")
```

```
# Trajectories in backward time.
ts = np.linspace(0, -4, 100)
ic = np.linspace(-1, 1, 5)
for r in ic:
    for s in ic:
        x0 = [r, s]
        xs = odeint(dx_dt, x0, ts)
        plt.plot(xs[:,0], xs[:,1], "r-")

# Label the axes and set fontsizes.
plt.xlabel("x",fontsize=15)
plt.ylabel("y",fontsize=15)
plt.tick_params(labelsize=15)
plt.xlim(-1,1)
plt.ylim(-1,1)

# Plot the vectorfield. See lines 10, 12 for system.
X,Y = np.mgrid[-1:1:10j,-1:1:10j]
u=a*X + b*Y
v=c*X + d*Y
pl.quiver(X,Y,u,v, color='b')
plt.show()
```

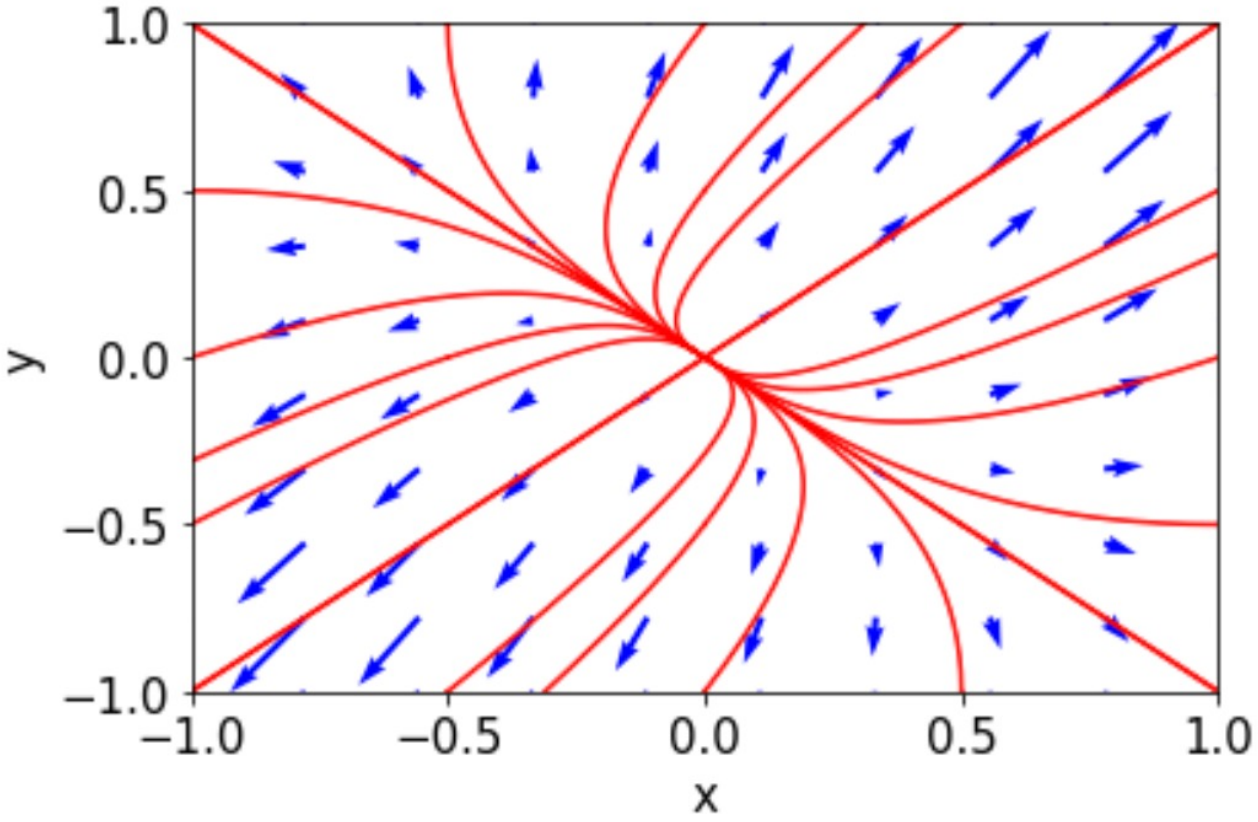
Plot trajectories for the system:

$$\frac{dx}{dt} = 2x + y$$
$$\frac{dy}{dt} = x + 2y$$

```
from sympy import *
A = Matrix([[2 , 1] , [1 , 2]])
A.eigenvals()
A.eigenvects()
```

Eigenvalues and eigenvectors:

$\lambda_1 = 1, \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ and $\lambda_2 = 3, \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.



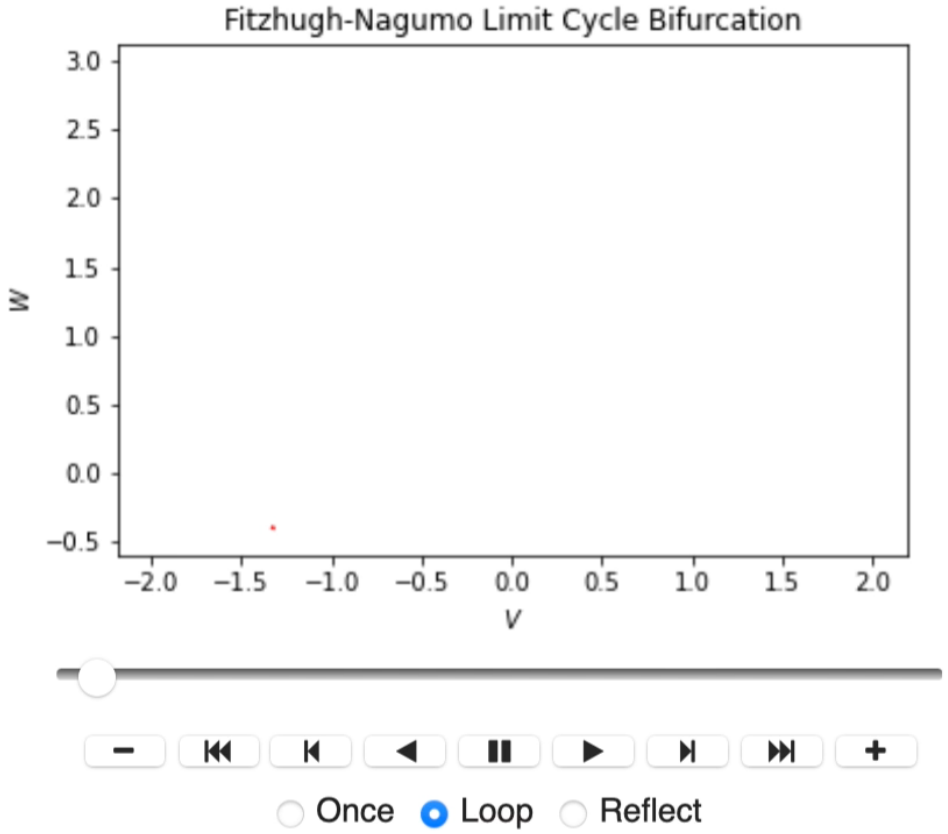
Week 3: Limit Cycle Bifurcations

Fitzhugh-Nagumo Model of a Neuron

$$\frac{dx}{dt} = x - \frac{1}{3}x^3 - y + I(\text{t})$$

$$\frac{dy}{dt} = \epsilon(x + a - by),$$

where $a = 1, b = 0.8, \epsilon = 0.08, 0.1 \leq I \leq 2.5.$



Week 5: Chua Circuit Animation

$$\frac{dv_1}{dt} = \frac{(G(v_2 - v_1) - f(v_1))}{C_1}, \frac{dv_2}{dt} = \frac{(G(v_1 - v_2) + i)}{C_2}, \frac{di}{dt} = -\frac{v_2}{L},$$

where v_1, v_2, and i are the voltages across C_1, C_2, and the current through L, respectively.

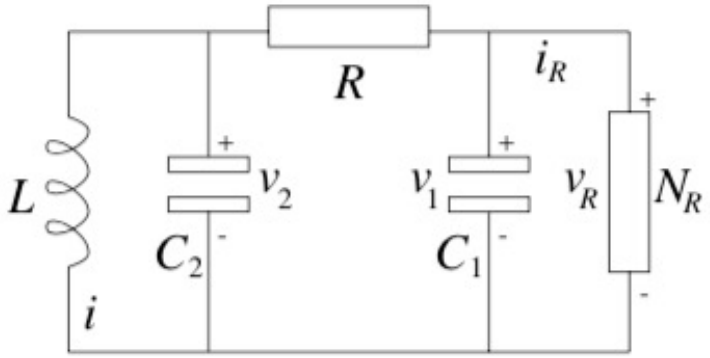
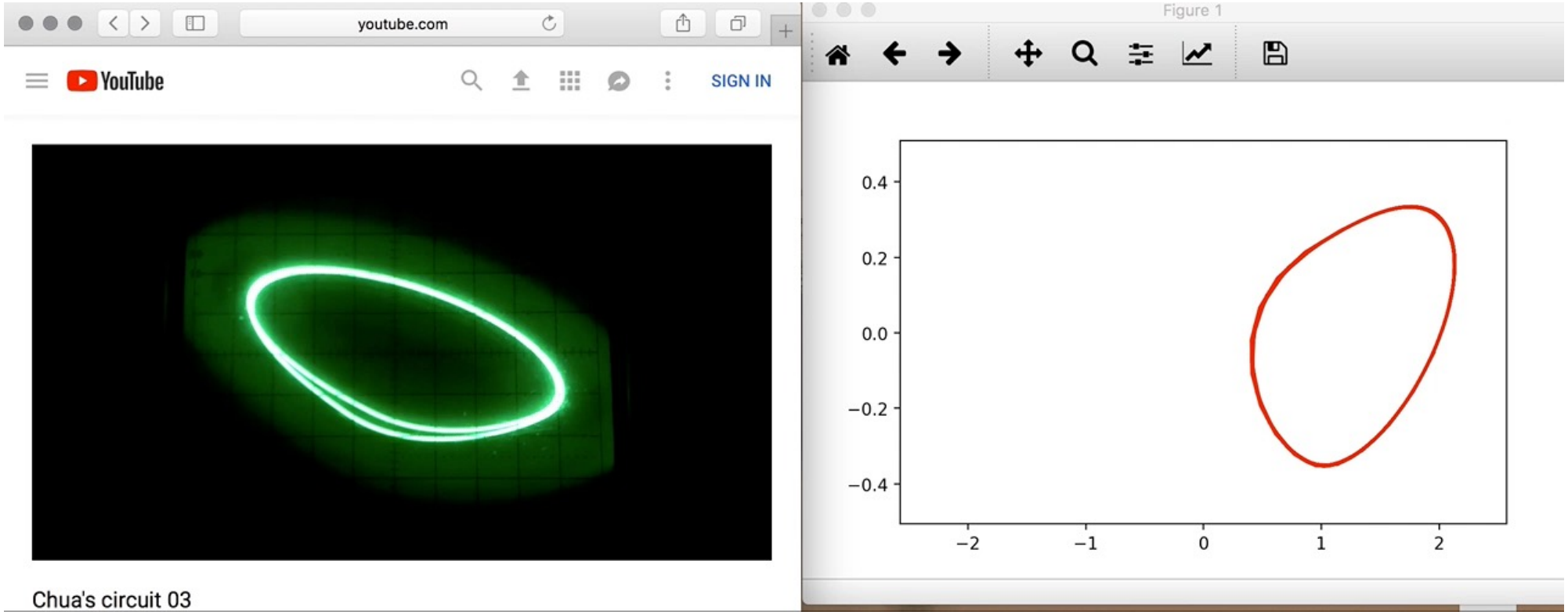


Figure 8.13: Chua's electric circuit.

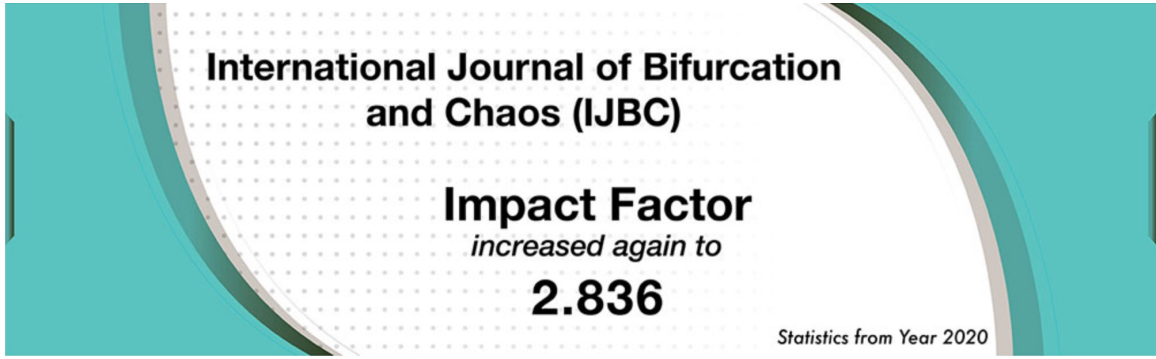
Week 5: Chua Circuit Animation



Undergraduate Publication

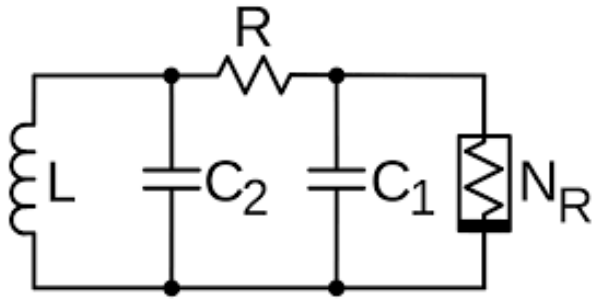


My co-inventor, Jon Borresen was my final year project student in 2001.

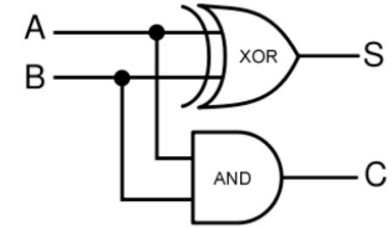


Editor: **Leon Chua**

Borresen J and Lynch S (2002) FURTHER INVESTIGATION OF HYSTERESIS IN CHUA'S CIRCUIT, *International Journal of Bifurcation and Chaos* **12**(1) 129-134.



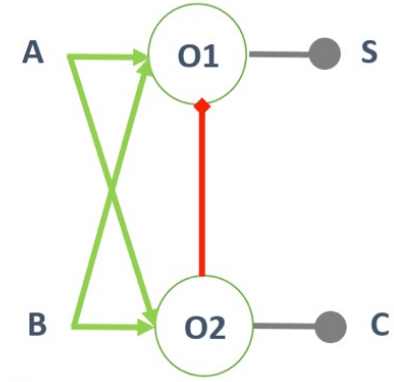
MMU Invention: Binary Oscillator Computing



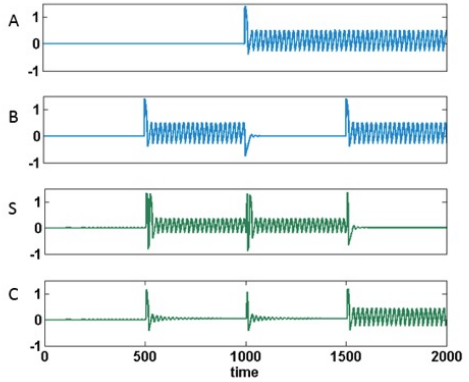
(a)

A	0	0	1	1
B	0	1	0	1
S	0	1	1	0
C	0	0	0	1

(b)

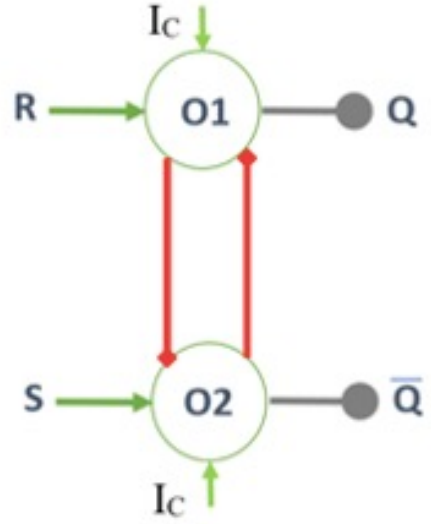


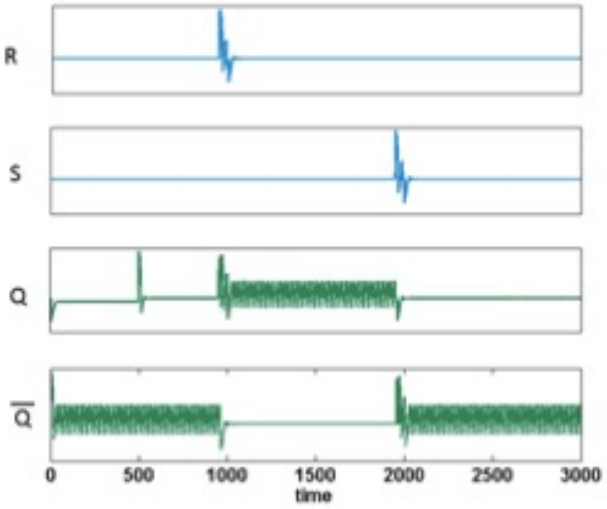
(c)



(d)

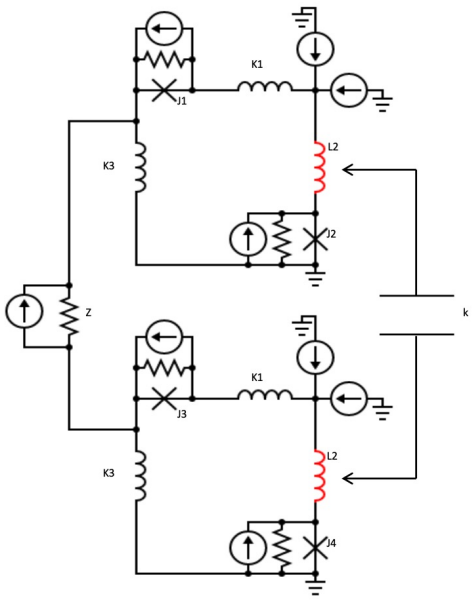
Binary Oscillator Half-Adder.

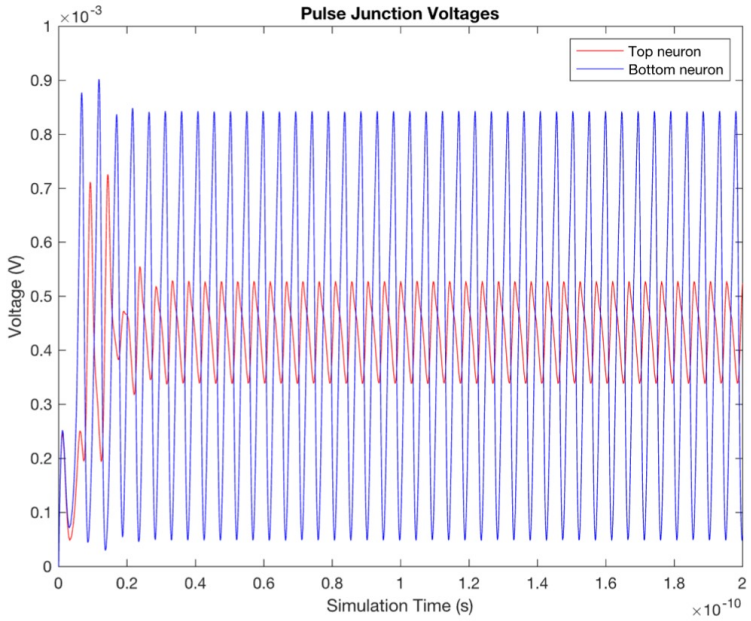


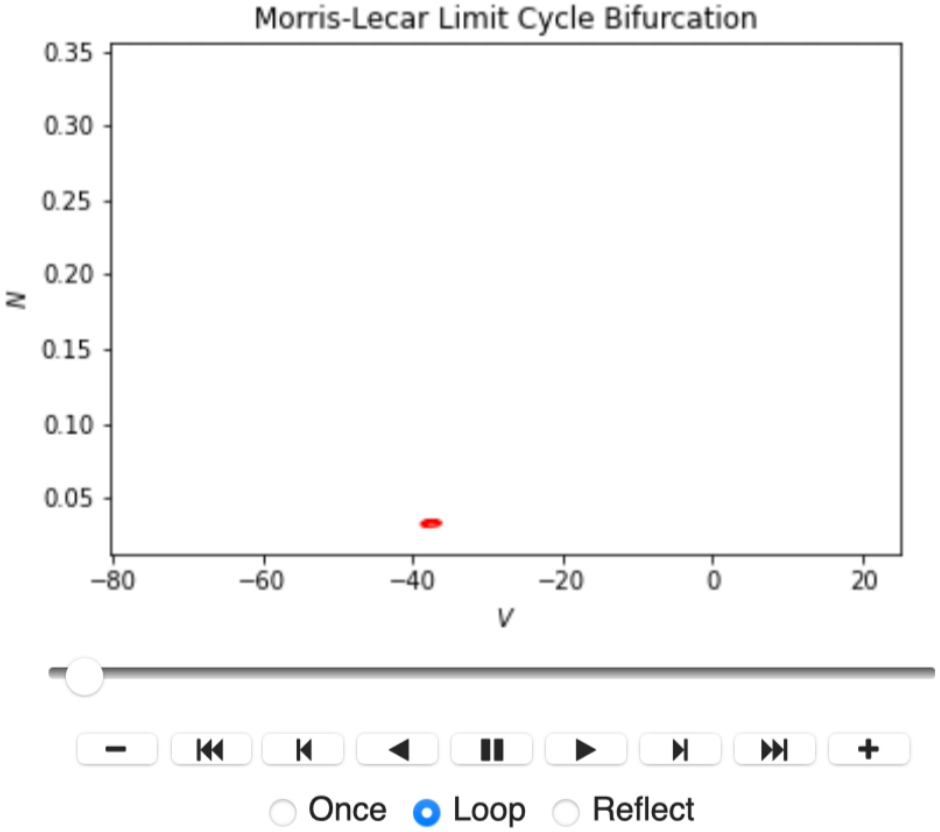


Binary Oscillator Set Reset Flip-Flop.

MMU Invention: Josephson Junction Set Reset Flip-Flop







Week 6: The Logistic Map $x_{n+1} = \mu x_n(1 - x_n)$

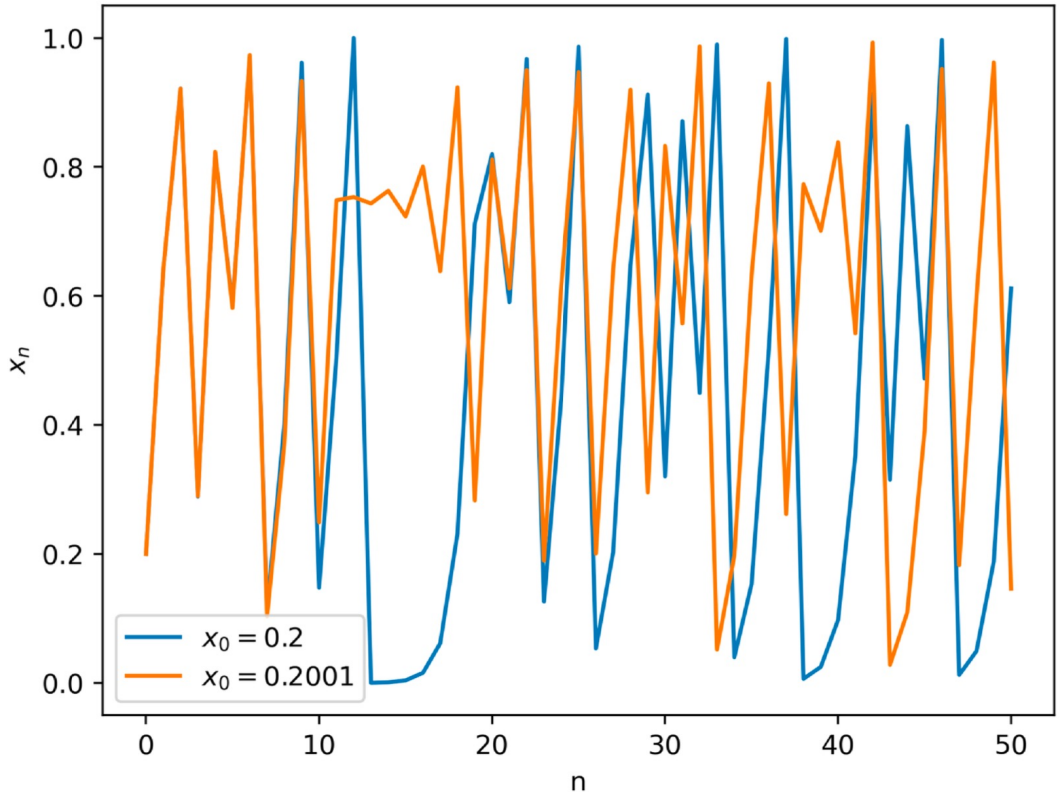


Figure: Sensitivity to initial conditions: chaos.

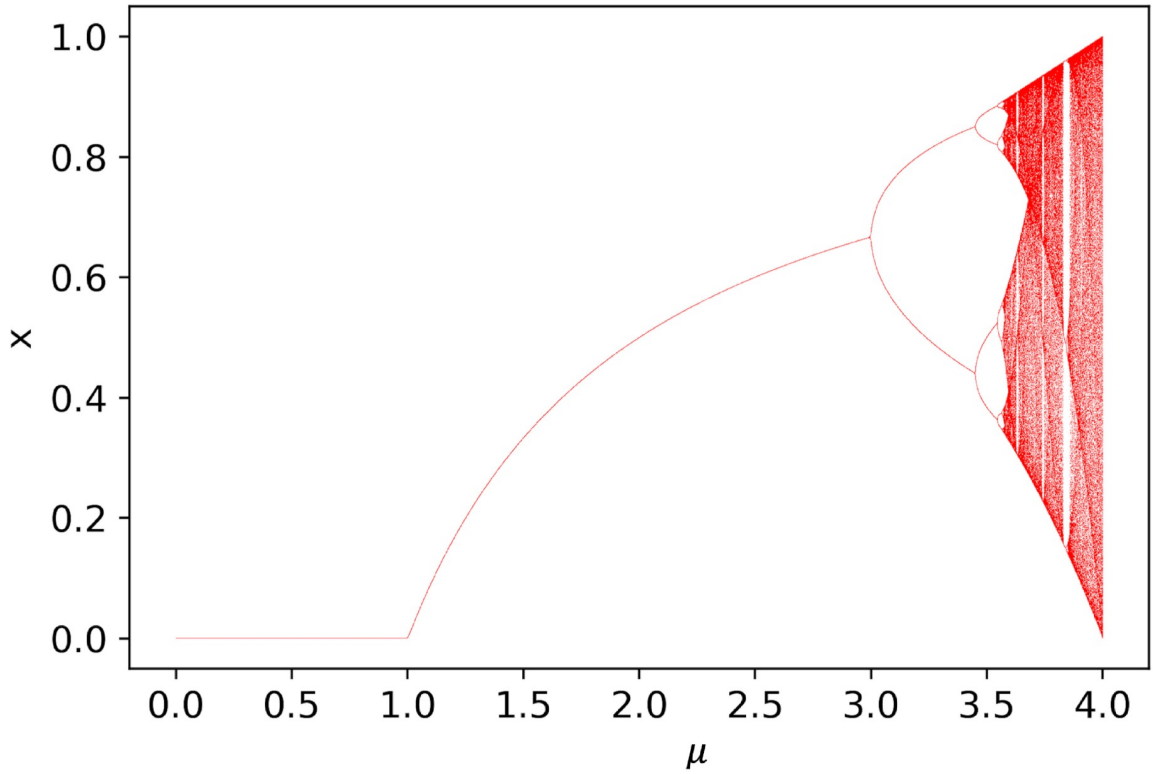


Figure: Bifurcation diagram.

Week 9: The Complex Ikeda Map $E_{n+1} = A + BE_n e^{i|E_n|^2}$

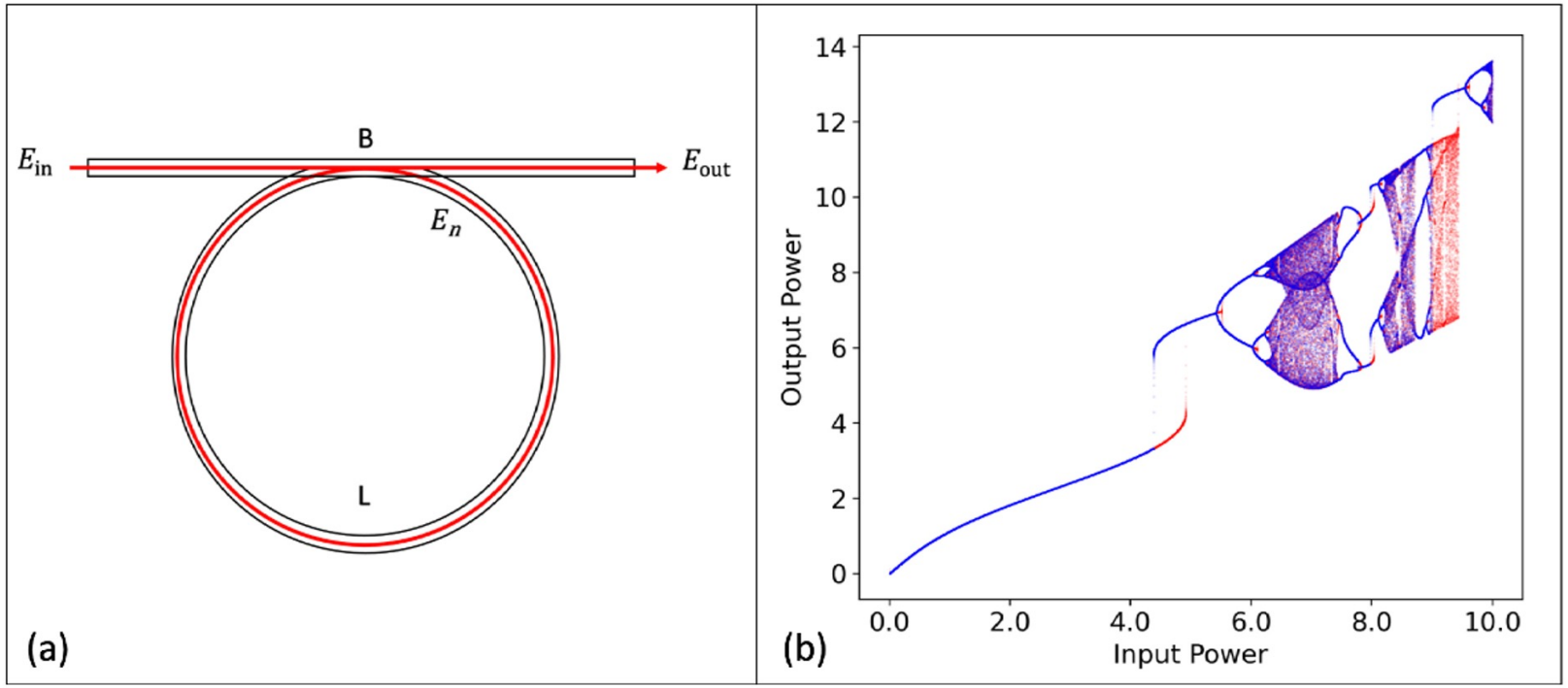


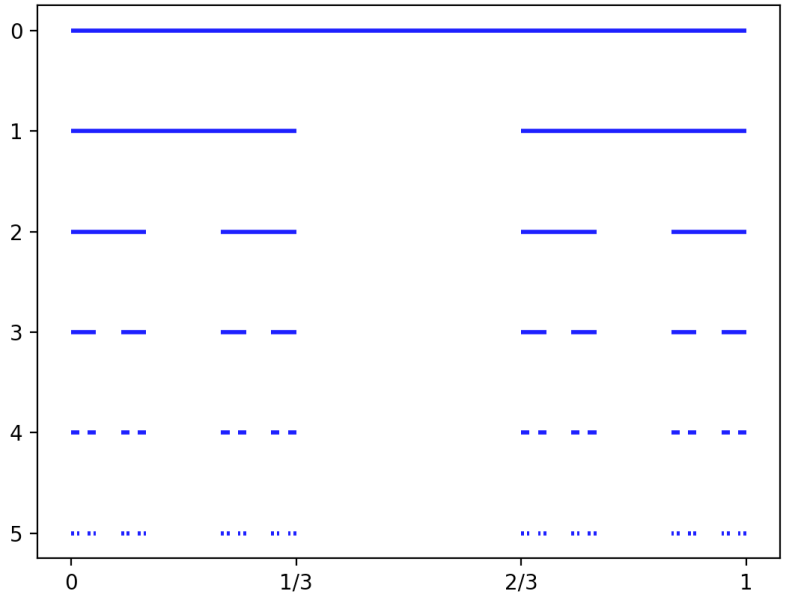
Figure: (a) The Simple Fibre Ring Resonator. (b) Bifurcation diagram using the second iterative method. Ramp up in red and down in blue. There is a hysteresis loop.

Week 10: Fractals and Multifractals

```python
# The Cantor Set.
import numpy as np
import matplotlib.pyplot as plt

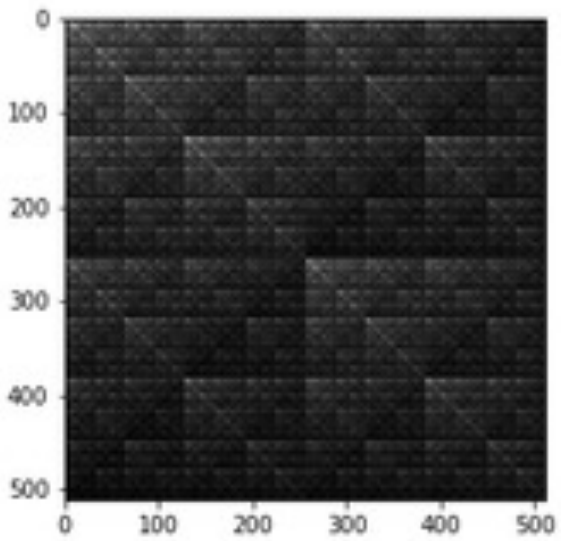
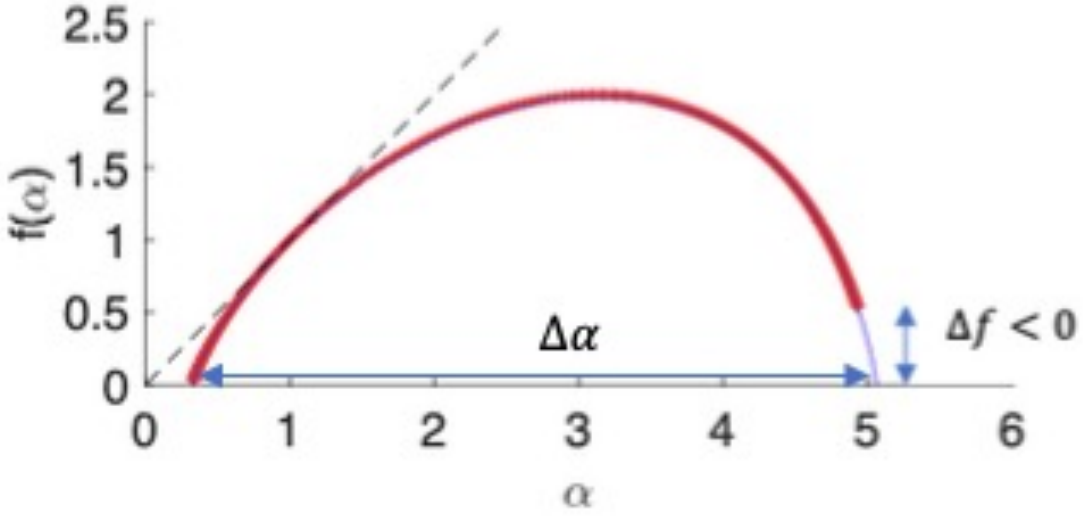
line, stage = [0 , 1] ,  5
def cantor(line, level = 0):
    plt.plot(line, [level ,level], color = "b", lw = 2,\
             solid_capstyle = "butt")
    if level < stage:
        segment = np.linspace(line[0] , line[1] , 4)
        cantor(segment[:2], level + 1)
        cantor(segment[2:], level + 1)

cantor(line)
plt.gca().invert_yaxis()
plt.xticks([0, 1/3, 2/3, 1],['0', '1/3', '2/3', '1'])
plt.show()
```



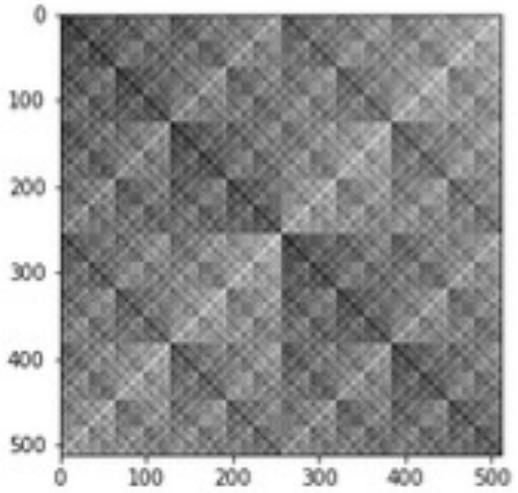
Problem: Edit the program to plot a Cantor set where the two middle fifth segments are removed at each stage.

Multifractals

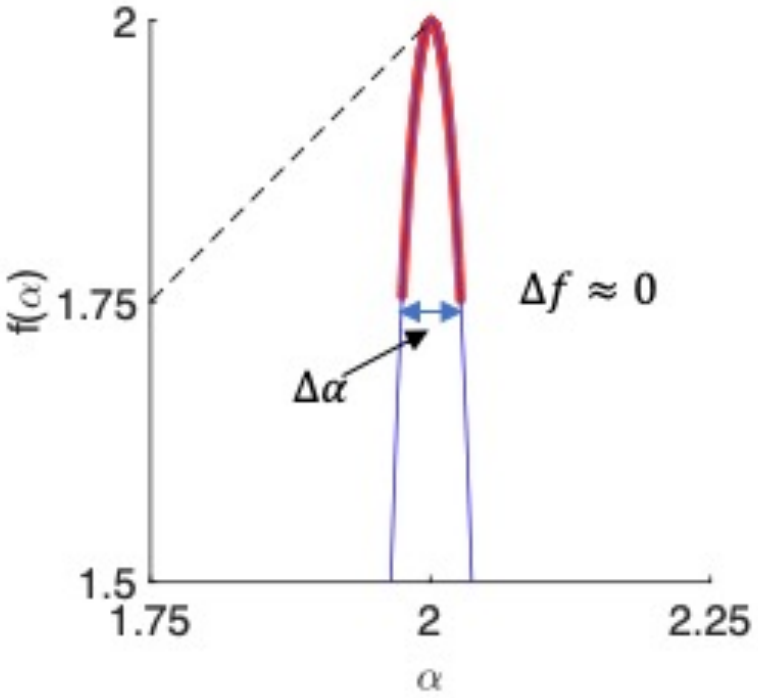
Multifractal motif	Representative figure	Multifractal f-α curve
0.8 0.1 0.03 0.07	 Clusters of dark pixels.	 The f-α curve is skewed left and $\Delta f < 0$.

Multifractals

0.24	0.26	
0.255	0.245	



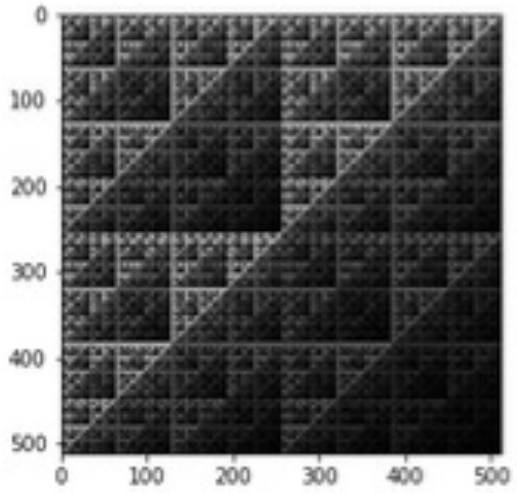
Homogeneous – no clusters.



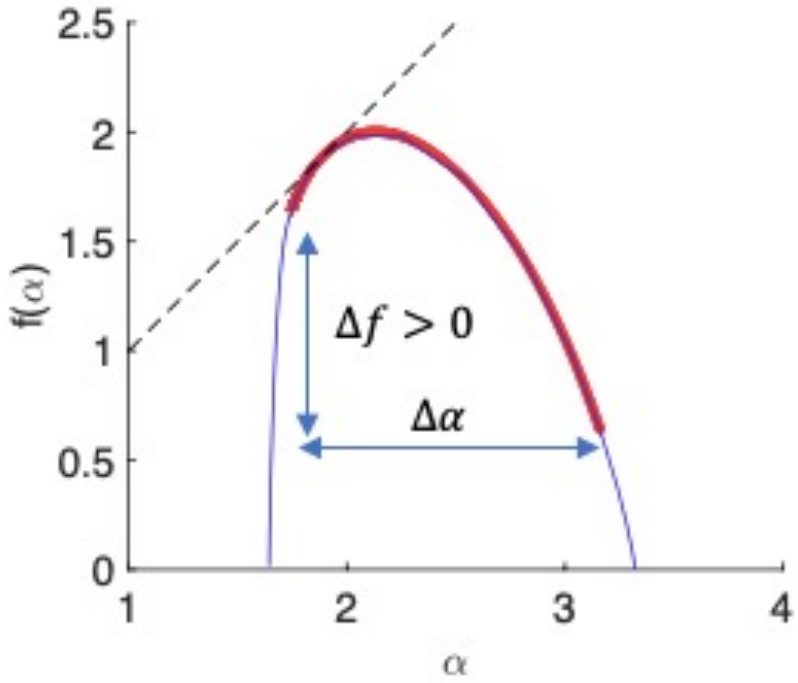
The f-α curve is not skewed and $\Delta f \approx 0$.

Multifractals

0.3	0.33
0.27	0.05



Clusters of bright pixels.



The f-α curve is skewed right and $\Delta f > 0$.

Multifractals Research

Wilson-Nieuwenhuis J, Taylor J, Gomes LC, **Lynch S**, Whitehead D, Whitehead KA (2024) Non-chlorine detergent formulations as an alternative for unpasteurised milk removal from stainless steel surfaces. Journal of Food Engineering 373, 112031.

Whitehead KA, **Lynch S**, Amin M, Deisenroth T, Liauw CM and Verran J. (2023) Effects of cationic and ionic surfaces on the perpendicular and lateral binding forces and attachment, adhesion and retention of Aspergillus niger conidia. Nanomaterials 13(22), 2932.

Evans A, Slate AJ, Tobin M, **Lynch S**, Wilson-Nieuwenhuis J, Verran J, Kelly P and Whitehead KA (2022) Multifractal analysis to determine the effect of surface topography on the distribution, density, dispersion and clustering of differently organised coccal shaped bacteria. Antibiotics 11(5) 11050551.

Whitehead KA, El Mohtadi M, **Lynch S**, Liauw CM, Amin M, Deisenroth T, Preuss A and Verran J (2021) Diverse surface properties reveal that substratum roughness affects fungal spore binding. iScience 24(4), 102333.

Slate AJ, Whitehead KA, **Lynch S**, Foster CW and Banks CE (2020) Electrochemical decoration of additively manufactured graphene macro electrodes with MoO2 nanowire: An approach to demonstrate the surface morphology, J. of Physical Chemistry C, 124(28) 15377-15385.

Wickens D, **Lynch S**, Kelly P, West G, Whitehead K, and Verran J, (2014) Quantifying the pattern of microbial cell dispersion, density and clustering on surfaces of differing chemistries and topographies using multifractal analysis, Journal of Microbiological Methods, 104, 101-108.

Mills SL, Lees G, Liauw C and **Lynch S** (2004) An improved method for the dispersion assessment of flame retardent filler/polymer systems based on the multifractal analysis of SEM images, Macromolecular Materials and Engineering, **289**(10), 864-871.

ResearchGate: http://www.researchgate.net/profile/Stephen_Lynch

ResearchGate Join for free Log in

Overall publications stats

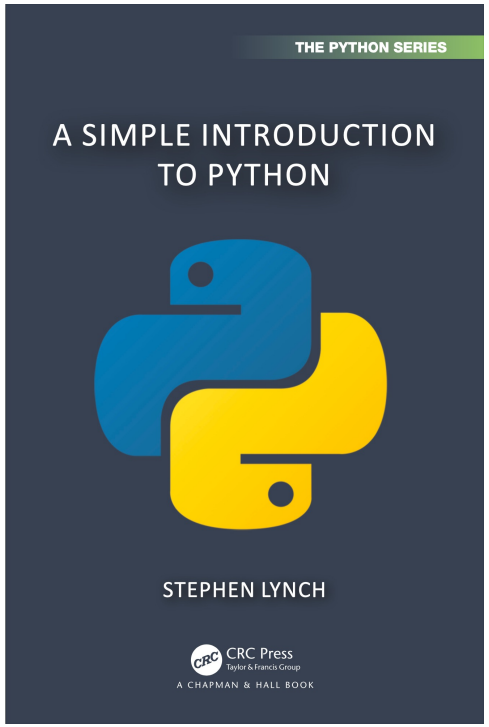


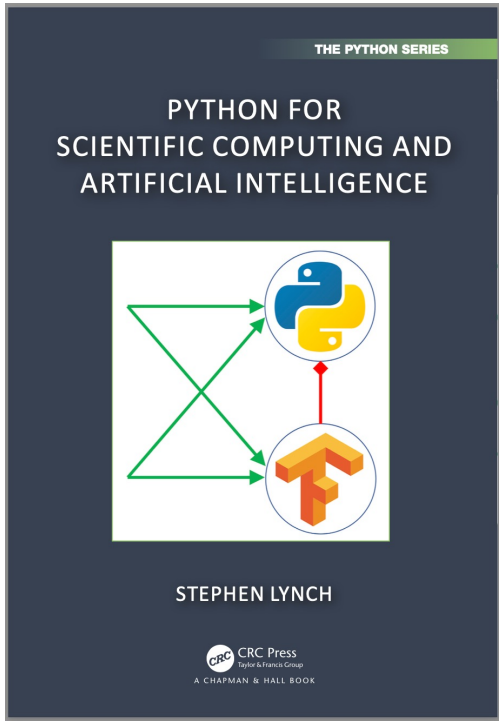
1,821	116,868	1,775	187
Research Interest Score	Reads	Citations	Recommendations
+1.5 last week	+167 last week	+1 last week	---

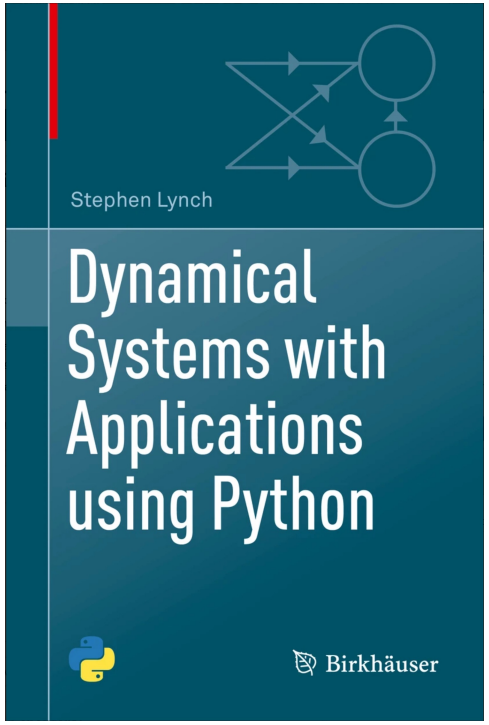
ScholarGPS:
https://scholargps.com/scholars/53907764717005/stephen-lynch?e_ref=5a87db3a668bc9f521b3

Stephen Lynch
ScholarGPS™ ID: 53907764717005

Highly Ranked Scholar - Lifetime:
#23 Dynamical system







THANK YOU FOR LISTENING! ANY QUESTIONS?